Determination of Best Fit Probability Distribution and Frequency Analysis of Threshold Rainfall under different Climate Change Scenarios

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Abstract
It is necessary to study and analyze the frequency of extreme rainfall events to determine the best-fit distribution that can predict the occurrence of the certain natural phenomena such as rainfall, flood, etc. In this study assessed to determine the best-fit distribution, the frequency analysis of threshold rainfalls considering Coupled Model Intercomparison Project phase 5 General Circulation Models (CMIP5 GCMs) under two Representative Concentration Pathways (RCP) scenarios (2.6 and 8.5). For this purpose, four empirical formulas (Hazen, Weibull, Tukey, and Cunnane) were used to estimate the return periods of threshold precipitation. Also, various probabilistic distributions including normal distributions, log normal (LN), log normal 3 (LN3), Gumble, Pearson type 3 (P3), and log Pearson type 3 (LP3) were applied to predict the distribution of threshold rainfalls. Kolmogorov-Smirnov test was used to determine the best-fit probability distribution function (PDF). Results revealed that the Hazen formula obtained the most estimate in the period of observation and future periods, and the near future (2015 - 2040) and the far future periods (2041 - 2065). According to the results, the LN3, LP3 and GEV probabilistic distributions presented the best PDF for threshold rainfalls in most periods. Among the best-fit distributions, LN3 was received 45 percent and LP3 and GEV received 20 and 30 percent of the best result, respectively. These results indicate there are severe abnormalities in the threshold precipitations, especially in high amounts. The results of this study can be used to develop more accurate models against the dangers, and damages caused by Extreme weather and flood.

Keywords: Tehran Province, Threshold Precipitation, Climate Change Scenarios, Frequency Analysis, Probabilistic Distribution

1. Introduction
Extreme rainfalls are important parameters affecting various natural and socio-economic systems like water resources management, watershed, agriculture, forestry, and tourism (Rosenzweig et al., 2001; Mailhot et al., 2012; Rosenberg et al., 2010; Murray and Ebi, 2012). Planning for emergency climatic situations such as design and building urban drainage systems, water resources management, reducing flood damages and pollution control requires sufficient knowledge of extreme weather with high risk of recurrence (Fikre, 2016). Therefore, frequency analysis is important to deal with
extreme events of rainfall and flood. (Serinaldi and Kilsby, 2015; Sun et al., 2017). Frequency of extreme rainfall has obviously increased, while light, and mild rains have fallen in warm areas of the world (Groisman et al., 2005; Moore et al., 2015; Zheng et al., 2016; Sun et al., 2017). The occurrence probability of extreme rainfall is less than 10% (Stocker, 2014) and the 95th percentile usually in percentage terms, is used as the threshold (e.g., Zhai et al., 2005; Zheng et al., 2016). The analysis of the frequency of rainfall helps to estimate the return periods, and the corresponding event values. The analysis of annual threshold rainfalls with return periods is also a basic tool for planning and designing dams, bridges, drainage works and determining drainage coefficients (Bhakar et al., 2008; Deraman et al., 2017; Yuan et al., 2017).

The assessment of possible changes in extreme precipitation in future periods has become one of the most important practical issues in analyzing hydrological risk, and design engineering because the accurate estimation of the frequency of extreme weather effectively reduces the effects of flooding through design, implementation, and utilization (Liu et al., 2015). The selection of the best-fit distribution of rainfall processes always plays an important role in hydrologic studies (Kang and Yusof, 2013). It is assumed that each hydrological variable has a specific distribution type (Aksoy, 2000). Some of the most important and commonly used probabilistic distributions functions (PDFs) that are used in hydrology include Normal distributions, Log-Normal (LN), Gumbel, Gamma 2 (G2), Pearson type 3 (P3), Log-Pearson type 3 (LP3) and Weibull (Aksoy, 2000; Khudri and Sadia, 2013). However, selecting an appropriate PDF is one of the major problems in engineering operations.

Several studies have been under taken on the frequency of extreme rainfall and the choice of appropriate distribution in the world. Ogunlela (2001) used five probabilistic distributions, namely Normal distribution, Normal log, Log Pearson type 3 and distribution of extreme value Type-1 for the analysis of the frequency of daily and monthly extreme rainfall events in Illinois, Nigeria. The results showed that LP3 is the best distribution for daily extreme rainfall, while the normal distribution described the monthly extreme precipitation in Ilorin better. Nadarajah and Choi (2007) analyzed the maximum annual daily rainfall during the years 1961-2001 in five, South Korean regions, and found that it is suitable to project extreme precipitation for the distribution of generalized extreme values in each region to predict their behavior in the future. They showed that Gamble distribution offers the best model for four out of five studying areas. Zin et al. (2009) used five tree-parameter extreme value distributions including generalized logistic (GL), lognormal (LN3) and Pearson (P3). They used these extreme distribution values to determine the most suitable distribution to describe the annual series of maximum daily rainfall from 1975 to 2004 in 50 rain gauge stations in the Malysian Peninsular. To identify the most suitable probable distribution, Fikre (2016) employed four probably distribution functions for estimating daily extreme rainfall series for different record periods of 18 rain gage stations from Bale zone of Ormia region, Ethiopia. The results showed that the Log Pearson type III is the best fitting for daily extreme rainfall. Onen and Bagatur (2017) have studied the prediction of flood frequency factor (K) for the Gumbel distribution using gene expression programming (GEP) and regression model. The results showed that the Gumbel distribution provided the best fit according to the extreme value analysis studies. The probability distribution of daily precipitation at the point and catchment scales was analyzed by Lei Ye et al. (2018). To examine distributional alternatives for the wet-day series probability plot correlation coefficients and L-moment diagrams are used. Their analysis indicates, both Pearson Type-III (P3) and kappa (KAP) distributions perform very well for point rainfall. Also, for wet-day precipitation, the KAP distribution best describes the distribution. Whereas, the performance of P3 distribution for wet-day precipitation at the catchment scale improved performance of fit over G2.
The main objective of this study is to determine the best fit probability distribution, and frequency analysis for annual threshold precipitation over Tehran province, Iran. The best fit probability distribution was also evaluated based on several suitable fit tests for an observational and future period. Information gained based on results are of several research programs, watershed management, hydrological and agricultural studies in Iran are inseparable.

2. Materials and Methods

2.1. Study area and data

Tehran Province with an area of 18,909 km² is located in the southern piedmont zone of Alborz mountain range of Iran (Fig. 1). It has a cold semi-arid climate with Mediterranean rain pattern and continental weather characteristics, at an altitude of 1178 meters above sea level. The average precipitation amounts to 250 mm which happens mainly in the winter, and spring. Tehran’s precipitation during a 50-year period indicates that the maximum and minimum precipitation in this province is fluctuates between 400 and 100 mm/year respectively (Jahani and Reyhani 2007). The Karaj and the Jajrood rivers and some other seasonal rivers, provide tap water for Tehran population (Iran Meteorological Organization, 2019). In the current study, selection of the stations was limited to four stations due to incomplete data in other stations. The data was then evaluated for their adequacy and functionality. As in the downscaling section, the statistical courses in the stations are different, but in the frequency analysis and statistical distributions sections, the statistical courses were selected jointly in all stations (1991-2017). In Table (1), the duration of recorded data and characteristics related to all the stations, is presented for accurate frequency precipitation analysis.

<table>
<thead>
<tr>
<th>Synoptic station</th>
<th>Duration</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Altitude above sea level (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abali</td>
<td>1986-2017</td>
<td>51.88</td>
<td>35.75</td>
<td>24.65.2</td>
</tr>
<tr>
<td>Geophysic</td>
<td>1991-2017</td>
<td>51.38</td>
<td>35.74</td>
<td>1418.6</td>
</tr>
<tr>
<td>Shemiran</td>
<td>1991-2017</td>
<td>51.48</td>
<td>35.79</td>
<td>1549.1</td>
</tr>
<tr>
<td>Mehrabad Airport</td>
<td>1961-2017</td>
<td>51.30</td>
<td>35.96</td>
<td>1191</td>
</tr>
</tbody>
</table>

![Fig. 1- Synoptic stations located in the study area](image)
2.2. Downscaling and Prediction

General Circulation Models (GCMs) are a kind of climatic model that are utilized for forecasting weather and climate change (Xu, 1999). These models are unable to show local sub-grid-scale dynamics and features (Wigley et al., 1990). The relations between large-scale predictors and local-scale predictions are determined using multiple linear regression models in Statistical Downscaling Model (SDSM) (Shukla et al, 2015). In this paper, the SDSM was applied to downscaling of rainfall from the GCM. The historical data of four synoptic stations and Global Climate Model data of CanESM2 under RCP2.6 and RCP8.5 were applied for the near (2015-2040) and far (2041-2065) periods. After calibration and validation with large-scale atmospheric variables encompassing the NCEP reanalysis data, the future period’s rainfall was forecast separately for the two RCPs.

2.3. Return period (T) analysis

The return period is the inverse of the probability that the event will happen in a year or is the average time interval for which an event (e.g. flood or river level) will occur once (Mays, 2010; Alam et al, 2018).

In the mathematical definition, if the variable (x) equals or be greater than an event of magnitude \( x_T \) and occurs once in \( T \) years, then the probability of occurrence \( P(X \geq x) \) of such a variable is expressed as:

\[
T = \frac{1}{P(x \geq x_T)} = \frac{1}{1-P(x \leq x_T)}
\]  

(1)

2.4. The correct probability positions for estimating return periods

An empirical distribution, supported a random sample from a probability distribution, is obtained by plotting the exceedance probability of the sample against the sample value. Several methods have been proposed for the determination of plotting positions. Most plotting position formulas are represented by Eq (3) (Chow et al., 1988).

\[
P(X \geq x_i) = \frac{(i-b)}{(n+1-2b)}
\]

(3)

Where \( i \) is the ordered rank of a sample value, \( n \) is the sample size, and \( b \) is a constant between 0 and 1, depending on the plotting method. Using different formulas for plotting position in equation (3) is presented in Table 2 (Chow et al., 1988).

2.5. Commonly Used Probability Distributions analysis

In this study, several types of probability distributions, including normal, lognormal, Log-Normal type 3, Pearson Type-3, Log Pearson Type-3, and Gamble were assessed to find out the best fit probability distribution of threshold rainfall. They are commonly used for probabilistic analysis of hydrology (Yuan et al, 2017). Fitting the theoretical probability distribution of the observed data was done by Weibull’s plotting position, (Tao et al, 2002). These PDFs is shown in Table (3) (Stedinger, 1993).

<table>
<thead>
<tr>
<th>Name of formula (Reference)</th>
<th>b</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hazen (Adeboye and Alatise, 2007)</td>
<td>( b=0.5 )</td>
<td>( P_i = (i-0.5/n) )</td>
</tr>
<tr>
<td>Weibull (Hirsch, 1981)</td>
<td>( b = 0 )</td>
<td>( P_i = (i/n+1) )</td>
</tr>
<tr>
<td>Tukey (Makkonen, 2008)</td>
<td>( b = 0.33 )</td>
<td>( P_i = (i-0.33)/(n+0.33) )</td>
</tr>
<tr>
<td>Cunnane (Cunnane, 1978)</td>
<td>( b = 0.4 )</td>
<td>( P = (i-0.4)/(n+0.2) )</td>
</tr>
</tbody>
</table>
Table 3- Probability distributions function for determination of best fit PDF

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability distribution function (PDF)</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>[ f(x) = \frac{1}{\delta \sqrt{2\pi}} \exp \left{ -\frac{(x-\mu)^2}{2\delta^2} \right} ]</td>
<td>(-\infty &lt; x &lt; +\infty)</td>
</tr>
<tr>
<td>Gumbel</td>
<td>[ f(x) = \frac{1}{a} \exp \left[ \pm \frac{x-b}{a} \right] \exp \left[ \pm \frac{x-b}{a} \right] ]</td>
<td>(-\infty &lt; x &lt; +\infty)</td>
</tr>
<tr>
<td>Pearson type 3 (P3)</td>
<td>[ f(x) = \frac{\lambda (x-\gamma)^{\beta-1} e^\gamma (x-\gamma)}{x \Gamma(\beta)} ]</td>
<td>(\gamma &lt; x &lt; +\infty)</td>
</tr>
<tr>
<td>Log-Pearson type 3 (LP3)</td>
<td>[ f(x) = \frac{1}{\beta} \frac{\ln(x) - \gamma}{\beta}^{a-1} \exp \left{ -\frac{\ln(x) - \gamma}{\beta} \right} ]</td>
<td>(0 &lt; x \leq e^\gamma ) (\beta &lt; 0)</td>
</tr>
<tr>
<td>Log-Normal type 3 (LN3)</td>
<td>[ f(x) = \frac{1}{\delta \sqrt{2\pi}} \exp \left{ -\frac{(x-\mu)^2}{2\delta^2} \right} ]</td>
<td>(\gamma &lt; x &lt; +\infty)</td>
</tr>
<tr>
<td>Log-Normal (LN)</td>
<td>[ f(x) = \frac{1}{\delta \sqrt{2\pi}} \exp \left{ -\frac{(x-\mu)^2}{2\delta^2} \right} ]</td>
<td>(x &gt; 0)</td>
</tr>
<tr>
<td>Generalized extreme value (GEV)</td>
<td>[ f(x) = \exp \left{ -\left[ 1 + \xi \left( \frac{x-\mu}{\delta} \right) \right]^{1/\xi} \right} ]</td>
<td>(-\infty &lt; \mu &lt; \infty ) (\delta &gt; 0)</td>
</tr>
</tbody>
</table>

2.6. Determination of the best fit PDF

In order to assess and compare and empirical and theoretical distribution model, the best distribution type tests are used for checking and contrast an empirical and theoretical distribution model used of Goodness-of-fit test statistics. This is accomplished by comparing the observed frequency, and the expected frequency of the theoretical distribution because some of the variables follow specific distribution (Tilahun, 2006). One of the most commonly used assessing for testing frequency distribution is the Kolmogorov-Smirnov Test (Stephens, 1974). The Kolmogorov - Smirnov test is a function of the greatest vertical distance between the theoretical and empirical distribution functions (Alam et al, 2018; Conover, 1999). The major purpose of this test is to compare the empirical cumulative frequency \(Z_{(i)} = F(x_{(i)}, \theta)\) with the Cumulative Distribution Function (CDF) of an assumed theoretical distribution \(F_n(x_{(i)})\). In other words, the K-S test calculates the maximum difference of the ordered data between the hypothesized distributions and empirical cumulative distribution function.

\[
D^+ = \max_i \left( i/n - Z_{(i)} \right) \quad (4)
\]

\[
D^- = \max_i \left[ - (i - 1)/n \right] \quad (5)
\]

\[
D = \max(D^+, D^-) \quad (6)
\]

2.7. Frequency analysis using frequency factors

A frequency analysis requires \(\mu\) (mean of annual threshold rainfall of observed and Predicted years), \(\delta\) (standard deviation of annual threshold rainfall of observed and Predicted years), a set of data (e.g., rainfall value series) and of the probability density function that best
describes the distribution of the data. The value of $x$ for any given $P$ (probability), or $T$ (return period), is to be calculated by the following formula (Chow et al. 1988; Sharma and Kumar, 2016):

$$x_T = \mu (1 + K_T C_v)$$  \hspace{1cm} (7)

Where $C_v$ is coefficient of variation ($\delta/\mu$) and $K_T$ is the frequency factor which depends on the $(T)$, the assumed frequency distribution and $\delta$.

The parameter $z$ is calculated by the following formula:

$$Z = W - \frac{2.515517 + 0.802853W + 0.010328W^2}{1 + 1.432788W + 0.189269W^2 + 0.001308W^3}$$  \hspace{1cm} (8)

$$D = \max (D^+, D^-)$$  \hspace{1cm} (9)

In Eq. (9) when $P > 0.5$, $1 - P$ is substituted for $P$.

$K_T$ for the Normal and Log-Normal type 3 is determined by the following formula:

$$K_T = Z$$  \hspace{1cm} (10)

Where

$$y_T = \bar{y} + K_T S_y \hspace{1cm} \bar{y} = \log x$$  \hspace{1cm} (11)

In the Pearson type 3 (P3) and Log-Pearson type 3 (LP3) distributions for $K_T$ calculated using:

$$K_T = Z + \left( Z^2 - 1 \right) K + \frac{1}{3} \left( Z^3 - 6Z \right) K^2 - \left( Z^2 - 1 \right) K^3 + ZK^4 + \frac{1}{3} K^5$$  \hspace{1cm} (12)

$$K = \frac{C_S}{6}$$  \hspace{1cm} (13)

$C_S$ is coefficient of skewness of data’s

Also in the Gumbel distribution, for $K_T$ are calculated which is most commonly used in the frequency analysis of large events:

$$K_T = \frac{\sqrt{6}}{\pi} \left[ 0.5772 + \ln \left( \frac{T}{T - 1} \right) \right]$$  \hspace{1cm} (14)

3. Results and discussion

3.1. Return period (T)

Figure (2) shows the return period $(T)$ in a year, showing annual rainfall thresholds with different formulas at selected stations for observation and future periods under climate change scenarios. According to the aforementioned form, the prediction of the return period in the year, in all regions and in all periods, under RCP2.6 and RCP8.5 scenarios, has the same results for each of the data along the X axis, while the amount of rainfall on the Y axis is different. At Abali station, the most extreme events in every 60-1100 years in the two far and near RCP2.6 and RCP8.5 near period have been used with data of 80-100 mm per hour based on the formula. The most extreme events during the far period of RCP8.5 and the observation period of this station occurred with data 60-80 mm per hour every 60-1100 years. Also at the Geophysics station, the most extreme events are expected to occur in every 60-1100 years in the near and far periods RCP2.6, respectively, in data 60-80 and 120-140 mm per hour, in the far period RCP8.5 of data 80-100 mm per hour, in the near period RCP8.5 and in the observation period of data 100-120 mm per hour. Also, the most extreme events in the Shemiran station in the near and far periods of RCP8.5 and far period of RCP2.6, with data from 80-100 mm per hour, in data near period RCP2.6, with data from 60-80 mm per hour, as well as in the observation period of the station with data 60-70 mm per hour occurred every 60-1100 year. But at the Mehrabad Airport station, in all periods of climate change scenarios and observation, the most extreme events occur in data 60-50 mm per hour every 60-1100 years.

The analysis of the return periods, based on different formulas, shows the greatest differences between the years of 60-1100 years. So that among all the formulas, the use of the Hazen formula has the highest estimate of the
return period and is consistent with the studies of Yuan et al. 2017, and then the formulas of Cunnane, Tukey and Weibull will be in all observations and future periods.

Fig. 2-Return period in years computed using four different formulas for different periods in Mehrabad Airport station.

3.2. Probability distribution function

In order to show the amount of rainfall and its intensity, selecting a suitable and stable distribution is a key step in Frequency analysis of rainfall (Hanson and Vogel 2008). According to Fig. 3, the number of days threshold rainfall in the studied stations at several observation and future periods is different under the scenarios of climate change. Geographic location and environmental factors in this area are the source of change in the threshold of rainfall. So, in the western regions of Tehran (Mehrabad airport station), the threshold of rainfall is lower than other stations. In the above figure, the probability distribution for annual threshold rainfall in Tehran province has been calculated during different observation and future periods using different PDFs. According to the results, it can be stated, using the results that the probability distribution suitable for each observation and future stations (the near periods and far of scenarios RCP2.6 and RCP8.5) varies.
3.3. Selection of the best fit PDF

Summary of statistical results of Kolmogorov-Smirnov test for determining the most suitable PDFs at selected stations during the observation and future periods under the climate change scenarios is presented in Table (4). According to the aforementioned table, it can be stated that among all distributions, LN3 had the best fit throughout the courses. Indeed, LN3 best-fit ted in 45% of the stations and periods examined to describe the threshold rainfall and predict their behavior. LP3 and GEV ranked second and third in terms of the best-fit distribution in the observation and future periods respectively. While in the studies of Zalina et al. (2002), Khudri and Sadia (2013) and Alam et al. (2018) GEV has shown best-fit probability distributed. In Fikre (2016), demonstrates best for daily rainfall analysis. Olofintoye et al. (2009) evaluated LP3 as the best-fit for analyzing the frequency of daily extreme rainfall. However, normal and LN tests were not best-fit at any of the stations and periods examined. Therefore, it can be concluded that the distribution of LN3 is the most suitable PDF for predicting threshold rainfalls in most of the stations and periods studied under climate scenarios in Tehran province.
Table 4: Kolmogorov-Smirnov (K-S) values for six different distributions in a case study with the best fit.

<table>
<thead>
<tr>
<th>Station Name</th>
<th>Periods</th>
<th>K-S Values</th>
<th>Best fit PDF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Normal</td>
<td>LN</td>
</tr>
<tr>
<td>Abali</td>
<td>RCP2.6- Near</td>
<td>0.21</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>RCP2.6- Far</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Near</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Far</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td>Geophysic</td>
<td>RCP2.6- Near</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>RCP2.6- Far</td>
<td>0.22</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Near</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Far</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Shemiran</td>
<td>RCP2.6- Near</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>RCP2.6- Far</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Near</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Far</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>Mehrabad</td>
<td>RCP2.6- Near</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>Airport</td>
<td>RCP2.6- Far</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Near</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>RCP8.5- Far</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>0.20</td>
<td>0.13</td>
</tr>
</tbody>
</table>

3.4. Frequency of probabilistic distributions

The observed and expected values calculated by various distributions for predicting threshold rainfall values in different regions and selected periods were calculated (Fig. 4). Here, matching the distribution of probability of the data observed by the distribution of Weibull. According to the aforementioned figure, various distributions at Abali and Shemiran stations during the far periods are estimated under climate change scenarios at high values of thresholds rainfall compared to the observed values that Pearson's Type 3 distribution is the most estimated compared to other distributions. Whilst in Geophysics and Mehrabad airport stations, the distributions showed, the high values of threshold rainfall during observation and future periods under RCP2.6 and RCP8.5 scenarios, in contrast Abali and Shemiran stations had a smaller estimate of observed values.

Also, the observed and expected value of observation values at the Abali station showed no change with the values calculated by different distributions. However, in studies by Yuan et al. (2017) the distributions examined showed a much lower estimate than the observed amount at very high values.
4. Conclusions
In general, the choice of a suitable model depends on the characteristics of the rainfall data available at the station under study. Consequently, in order to select a best-fit, various distributions should be investigated so that accurate estimate and analysis of the threshold rainfall can be obtained (Khudri and Sadia, 2013). In this study, six probabilistic distributions were used to determine the best-fit probability distribution and five experimental formulas for frequency analysis of threshold rainfall precipitations at four weather stations of Tehran province during the observation and future periods under climate change scenarios. Also, the Kolmogorov-Smirnov test (K-S) was used to select a best-fit distribution model. The analysis of threshold rainfall during the return periods (which is important for determining long-term risks) using the Cunnan, Tukey, Hazen, and Weibull formulas showed that the return period for high-value rainfall varies in different formulas. Among all the formulas, the use of the Hazen formula is the highest estimate and the Weibull formula is the least estimate in the return period. Analysis and evaluation of threshold rainfall influence the design and construction of floodplain structures to alleviate the effects of flood influences. Therefore, the assessment of
such events helps to advance science to reduce and prevent the effects and damage caused by floods (Norbiato et al., 2007). Unlike Geophysics stations and Mehrabad stations under the climate change scenarios, the frequency of threshold rainfall at Abali and Shemiran stations shows that the different distributions used from extreme threshold rainfall are relatively higher than the observed values.

Selection of best-fit distributions using Kolmogorov-Smirnov test for different regions was calculated. However, among all distributions, LN3, LP3, and GEV had the best-fit distribution. Indeed, LN3 alone made up 45%, and LP3 and GEV each gained for 30% and 20% of the total distribution, respectively, as the better-fit. These results indicate that there are severe abnormalities in the threshold rainfall, especially in high quantities.

The results of this study can be used to develop better models against the dangers and damages caused by happening extreme rainfall and flood events. The data obtained for several research programs like watershed studies, hydrological studies, and agricultural studies in Iran are inseparable. These estimates can help managers and policy makers apply their initiatives and instructions to save people's lives and property of the people. Additionally, understanding the threshold rainfall pattern of a wide range of applications in hydrology, engineering design, climate studies, the environment, and agriculture will be helpful.

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6- Disclosure statement
No potential conflict of interest was reported by the authors.

7- References


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