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ORIGINAL ARTICLE

Investigation the Joint Probabilistic Behavior of River Flow Discharge-Suspended Sediment Load in Nazlochai Basin, Urmia

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Abstract

Accurately determining the relationship between river flow discharge and suspended sediment load in watersheds is challenging due to the influence of natural and human-induced variables on river estimations. Therefore, it is essential to employ modern methods to improve existing models. In this context, multivariate methods and copula-based modeling and simulation, given their ability to analyze data distributions, can be suitable options. In this study, joint frequency analysis of river flow discharge and suspended sediment load was conducted at the Abajalo and Tapik stations in the Nazlochai sub-basin, Lake Urmia, Iran using copula functions and marginal distributions. First, the correlation between variables was examined using Kendall's tau coefficient, indicating a strong positive relationship between river flow discharge and suspended sediment load. For modeling marginal distributions, the Log-Normal and GEV distributions with NSE=0.99 at the Abajalo station were selected as the best distributions for flow discharge and suspended sediment load, respectively, while the GEV and Generalized Pareto distributions with NSE=0.99 at the Tapik station were chosen for river flow discharge and suspended sediment load, respectively. In the joint analysis, the Galambos and Gumbel-Hougaard copula functions demonstrated the best performance based on evaluation criteria. Bivariate analysis revealed that at the Abajalo station, with a 90% probability and river flow discharge exceeding 40 m³/s, the suspended sediment load reaches over 3,000 tons/day, while at the Tapik station, the same probability with a river flow discharge of 25 m³/s indicates a suspended sediment load exceeding 500 tons/day. Finally, based on the conditional density of copula functions and considering various probabilities, equations were proposed for simulating suspended sediment load conditioned on river flow discharge at both stations. The proposed equations were suggested for probability levels of 80–90%, 90–95%, and 95–99% and evaluated using various statistical metrics. The proposed equations for conditional estimation of suspended sediment load demonstrated high performance at the Abajalo station (NSE>0.95 and 386.2<RMSE<642.8 tons/day) and the Tapik station (NSE>0.84 and 26.8<RMSE<836.8 tons/day). This study emphasizes that multivariate methods based on copula functions are effective tools for modeling nonlinear hydrological relationships.

Keywords: Conditional density, Conditional simulation, Copula function, Joint probability.

1. Introduction

Soil erosion, sediment transport, and predicting sediment amounts in rivers are fundamental challenges in water resource management. These phenomena not only cause financial losses but also affect the lifespan of dams, irrigation networks, and other water facilities. Accurate prediction of these processes can lead to better planning and reduced problems. River flow discharge plays a key role in sediment transport. With population growth and increasing demand for water, as well as recent climate changes, the importance of studying river behavior has increased. Decreased river flow discharge is influenced by various factors such as changes in rainfall patterns, human activities, and drought. This reduction can have widespread consequences for the environment (Khalili et al., 2016, Tahroudi et al., 2019). Statistical analysis of this parameter along with its joint estimation can provide accurate information about its changes and nature.

Statistical analysis of extreme values such as floods and low flows has a long history in hydrological research, and various researchers have employed different statistical distributions to model this phenomenon. Key studies in this area include:

Chow (1954) was a pioneer in this field, who first introduced the three-parameter log-normal distribution for estimating low flow quantiles. This method served as a foundation for the development of subsequent models. Matalas (1963) in a comprehensive study of 34 Canadian watersheds, showed that the Weibull distribution provided the best fit to minimum river flow discharge data. These findings formed the basis for many subsequent investigations. Loganathan et al. (1985), by comparing different distributions, concluded that the log-Pearson Type III and Weibull distributions exhibited high accuracy in modeling minimum flows. Landwehr et al. (1987), utilizing an innovative resampling method, compared the performance of four probability distributions. The results showed that the three-parameter Weibull and three-parameter log-Pearson distributions had good capabilities in modeling annual 7-day minimum flows.

These studies indicate that the selection of an appropriate distribution for analyzing low flows depends on various factors, including the hydrological conditions of the region, the length of the statistical period, and the analysis method. In recent years, combining these classical methods with new techniques such as neural networks and hybrid models has significantly increased the accuracy of analyses. Many studies exist regarding the analysis of various hydrological events in a univariate setting, but considering the nature of hydrological variables and their dependence on other existing parameters, the need for multivariate analysis becomes apparent.

Multivariate analysis of hydrological parameters is recognized as an advanced approach in water resource studies, rooted in the research of Snyder (1962) and Wong (1963). These researchers laid the initial foundations for the use of multivariate methods in hydrological data analysis. Considering various studies, the evolutionary path of multivariate methods can be described as follows:

1. Early Period (1960s):
   - Focus on simple bivariate models
   - Initial application in simultaneous rainfall-runoff analysis
   - Development of basic concepts in the interdependence of hydrological parameters
2. Development of Analytical Methods (1980s-2000s):
   - Introduction of advanced bivariate distributions including:
   - Bivariate normal distribution
   - Bivariate exponential distribution
   - Bivariate gamma distribution
   - Bivariate extreme value distribution
   - Widespread application in flood and drought frequency analysis (Durrans et al., 2003; Shiau, 2003)
3. Modern Research (2000s onwards):
   - Development of hybrid models using copula functions
   - More comprehensive analyses considering multiple parameters simultaneously
   - Application of non-parametric methods in multivariate analysis (Beersma and Buishand, 2004; Nadarajah and Gupta, 2006)

Multivariate analysis of hydrological parameters has transformed from a simple theoretical method into a powerful tool in water resource management. As demonstrated by Wang (2016) and He et al. (2007), these methods provide a more comprehensive understanding of the complex relationships between hydrological parameters. The continuous development of these methods, particularly through the use of copulas, has opened up new horizons in predicting and managing hydrological risks.

Sklar (1959) revolutionized the multivariate analysis of hydrological phenomena with the introduction of the concept of the copula function. This method, overcoming the limitations of traditional methods, allows for the joint modeling of variables with different marginal distributions. Unlike classical bivariate distribution functions, which required the variables to have the same type of distribution, copulas offer unparalleled flexibility. The key advantages of copulas are as follows: 1) the ability to combine heterogeneous marginal distributions, 2) separation of the dependence structure from the marginal distributions, 3) the ability to model nonlinear and complex dependencies, and 4) the possibility of simultaneously analyzing more than two variables.

De Michele and Salvadori (2003) were among the first researchers to apply this method in rainfall frequency analysis. This study paved the way for broader applications of copulas in water sciences. Salvadori and De Michele (2004) continued this path by utilizing copulas for flood frequency analysis. In Iran, various researchers have contributed to the development of these methods, including: Sanikhani et al. (2014): joint modeling of temperature and precipitation, Abdi et al. (2016): comprehensive drought analysis, Kavianpour et al. (2018): assessment of drought characteristics in various Iranian cities, Nazeri Tahroudi et al. (2020): joint frequency analysis of meteorological and hydrological drought, Nazeri Tahroudi et al. (2022): joint frequency analysis of water resource scarcity signals, Nazeri Tahroudi et al. (2023): statistical downscaling, whose research results showed that copulas have a better ability to identify extreme states and provide more reliable results.

Shiau and Lien (2021) presented a probability-based model using copulas for modeling daily suspended sediment load using river flow discharge data in Taiwan. They first constructed a bivariate distribution model based on copulas from sediment and river flow discharge data and used the conditional distribution of suspended sediment with respect to the observed river flow discharge to provide probabilistic estimates of sediment loads. Furthermore, four different methods based on the derived conditional distribution of suspended sediment were used to provide single-value estimates. Their research results were compared with those associated with the traditional sediment rating curve and evaluated using the root mean square error (RMSE), mean absolute percentage error (MAPE), Nash-Sutcliffe efficiency (NSE), and modified Nash-Sutcliffe efficiency (MNSE). The results showed that the copula-based methods performed better in MAPE and MNSE. Furthermore, the scattered characteristics of the observed suspended sediment load-river flow discharge relationships were preserved in the copula-based method and exhibited a similar frequency distribution to the recorded sediment data.

Yang et al. (2023) proposed a bivariate joint distribution framework based on nonparametric kernel density estimation (KDE) and a hybrid copula function to describe the complex river flow discharge-sediment dependence structure. In this framework, nonparametric KDE was used to fit the marginal distribution functions of the river flow discharge and sediment variables, and then a hybrid copula function was constructed by combining Clayton, Frank, and Gumbel copulas and compared with five common single copulas (Clayton, Frank, Gumbel, Gaussian, and t). Their research results showed: (1) Compared with the Gamma and Generalized Extreme Value (GEV) parameter distributions, the marginal distribution functions of the river flow discharge and sediment variables can be effectively obtained based on nonparametric KDE. (2) Compared with a single copula, the hybrid copula more completely reflects the complex dependence structure of the river flow discharge and sediment variables. (3) Compared with the best single copula, the return period accuracy based on the hybrid copula can be increased by 7.41%. Furthermore, the joint probability of river flow discharge and sediment in JRB is 0.553, and the disjoint probability of river flow discharge and sediment is 0.447. This study not only can improve the accuracy of statistical analysis of river flow discharge and sediment in JRB, but also provides a useful framework for other bivariate joint probability analyses.

Vahidi et al. (2024) used copulas to estimate the joint probability and joint return period of suspended sediment load (SSL) with respect to rainfall and river flow discharge values. In this study, the Vine copula family was used for the three-variable frequency analysis of SSL related to rainfall and river flow discharge in a watershed in Iran. They used daily data from 140 recorded events at a hydrometric station over the years 1975 to 2020.

To create the probabilistic model, D-vine, C-vine, and R-vine copula structure configurations, as well as rotational, Gaussian, and independent states, were examined based on the log-likelihood, AIC, and BIC criteria. The results of their research showed that the marginal distributions showed that the generalized Pareto (RMSE=3.91, NSE=0.98, MAPE=3.91), log-normal (RMSE=2.69, NSE=0.99, MAPE=7.69), and GEV (RMSE=2.16, NSE=0.99, MAPE=5.43) distributions were the best-fitted distributions to the rainfall, river flow discharge, and SSL data, respectively. By examining different Vine structures, D-vine was selected as a suitable copula (AIC=−727.8, BIC=−714.3, and log-likelihood=366.9) for modeling the dependence structure between rainfall (R), river flow discharge (Q), and SSL. They stated that the return period provided is conditional on runoff and rainfall occurrences. Compared to the univariate method, the provided return period includes a wider range of data.

Zhao et al. (2025) modeled the dependence between peak flow discharge, flood volume, and flood duration using Vine copulas and identified the nonlinear dependence structure between flood characteristics by comparing the performance of 5 different copula families, including Frank, Clayton, and Gumbel. They developed a new model for estimating the multivariate flood return period.

Copulas, as a novel paradigm in the multivariate analysis of hydrological phenomena, continue to be developed and evolve more than six decades after their initial introduction. These methods, by providing a flexible and accurate framework, have facilitated a better understanding of the complexities of hydrological systems. As numerous studies have shown, the application of these methods in real-world conditions has led to significant results in water resource management and the prediction of extreme events.

Nazeri Tahroudi et al. (2025) investigated two hybrid methods – VAR-GARCH and Copula-GARCH – for simulating suspended sediment load (SSL) using river flow discharge data from the Deh Mola station (Zahreh River basin). Their research results showed that the VAR-GARCH model demonstrated strong performance and achieved a Nash-Sutcliffe efficiency (NSE = 0.99) in SSL simulation, although some residuals fell outside the 95% confidence interval. In contrast, the Copula-GARCH model, fitted to the VAR-GARCH residuals, significantly improved accuracy and reduced RMSE to 1455 tons/day – a 66% reduction in error (improvement rate: 70%).

Increased intense rainfall in recent years has led to numerous floods and, consequently, an increase in the volume of sediment transported by rivers. Studies show that in many regions of Iran, rainfall amounts have a decreasing trend. To reduce the damage caused by floods and sediment, and to design appropriate water structures, it is necessary to accurately analyze parameters such as maximum river flow discharge and suspended sediment load.

Various methods exist for this analysis, each with its own advantages and limitations, and the choice of the best method depends on the conditions and available data. Improving the accuracy and efficiency of models is always a focus in hydrological modeling.

In this research, a method for predicting sediment based on flood discharge is presented using river flow discharge and suspended sediment data, based on marginal distributions and marginal behavior. This method can help to more accurately simulate the sediment carried by rivers.

Based on a review of the literature, it can be observed that the performance of copulas in simulating and analyzing the frequency of various hydrological events in different regions has been confirmed. Therefore, in this study, an attempt was made to analyze the joint frequency of the paired variable of river flow discharge-suspended sediment load at hydrometric stations located in the Nazlochai sub-basin, Lake Urmia basin, Iran, by utilizing bivariate copulas.

2. Materials and Methods

In this study, river flow discharge data (m^3/s) and corresponding suspended sediment load (tons/day) data were used at two hydrometric stations, Abajalo and Tapik, in the Nazlochai sub-basin, Lake Urmia basin, Iran. The data used at the Abajalo station include 153 events in the statistical period 2001-2023, and at the Tapik station, the data include 359 events in the statistical period 1990-2023. Figure 1 shows the location of the study sub-basin. Table 1 also shows the statistical characteristics of the data under investigation during the mentioned statistical period.

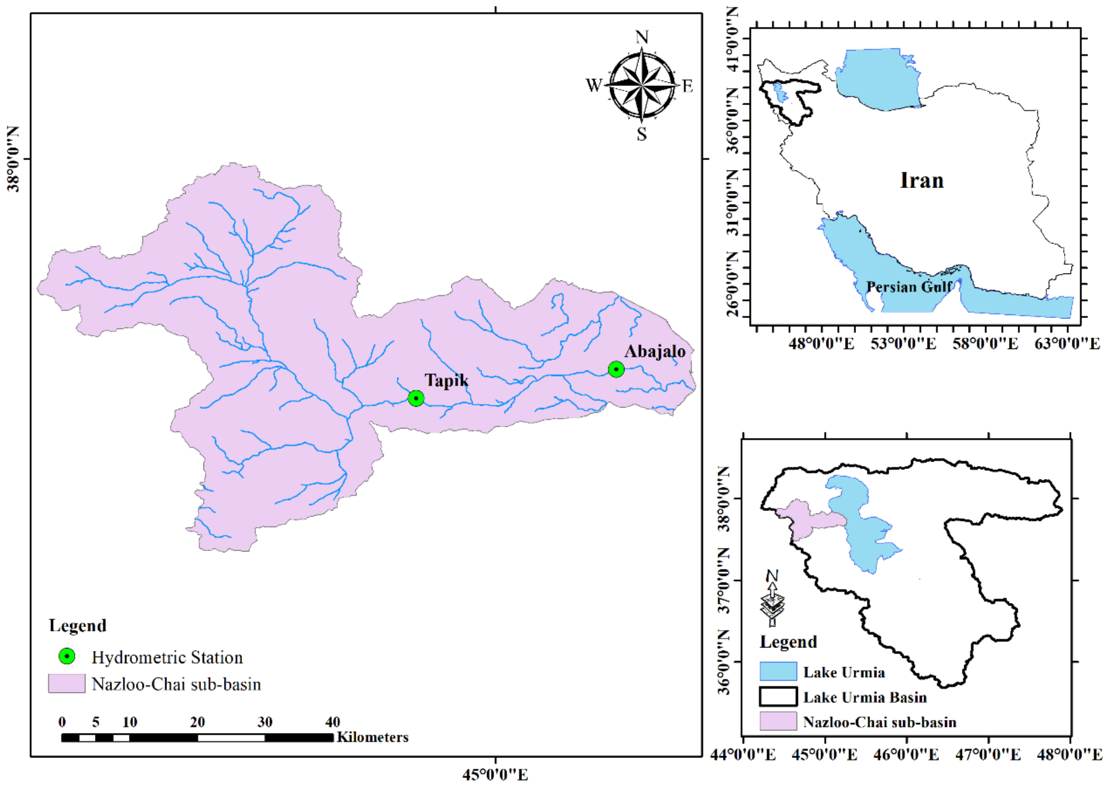


Fig. 1. Location of the Study Area

Table 1. Statistical Characteristics of Data Studied in the Nazlochai Sub-basin, Urmia

Station	Variable	max	std	var	skew	average
Abajalo	River flow discharge (m^3/s)	107.0	16.7	279.7	2.7	11.6
	Suspended sediment load (ton/day)	30524.9	3793.6	14391131.4	5.0	1411.4
Tapik	River flow discharge (m^3/s)	251.5	19.6	383.6	6.9	10.9
	Suspended sediment load (ton/day)	32384.3	2736.9	7490450.5	8.8	536.3

2.1. Requirements for performing frequency analysis using copulas

In multivariate frequency analysis using copulas, before estimating these functions, two essential steps must be performed. The first step is to assess the degree of dependence between the variables under investigation, which is generally measured using the Spearman correlation coefficient and Kendall's tau statistic. If there is no significant correlation between the variables, the use of copula-based methods will not be feasible.

Therefore, the existence of a strong correlation between the data is considered a fundamental condition for performing multivariate analyses. Based on the findings of this research, the dependence between variables is significantly confirmed, so we can be highly confident in the accuracy of the results obtained from the bivariate analyses. Numerous studies, including the research of Bedford and Cooke (2001), Kurowicka and Cooke (2007), Aas et al. (2009), Gräler et al. (2013), Cooke et al. (2015), Bezak et al. (2017), Bevacqua et al. (2017), Zhang et al. (2018), Khan et al. (2019), Brunner et al. (2019), and Ramezani et al. (2019), have also suggested the use of Kendall's tau statistic as a suitable measure for assessing the correlation between variables.

The second step is to determine the appropriate marginal distribution for the data under study. In this stage, various probability distributions are tested on the data, and then the best model is selected based on evaluation criteria. For this purpose, two main indices, the root mean square error (RMSE) and the Nash-Sutcliffe efficiency (NSE), are used, the details of which are provided in the Model Assessment Criteria section. The distribution with the highest NSE value and the lowest RMSE value will be selected as the optimal distribution for analysis. This approach has also been validated in studies such as Bedford and Cooke (2001), Aas et al. (2009), and Brunner et al. (2019). These steps are executed step-by-step to ensure that the copula-based frequency analysis is performed with the necessary accuracy and validity.

2.2. Copula Functions

The concept of copulas was first introduced by Sklar (1959). These functions, by combining the marginal univariate distribution functions, allow for the construction of a multivariate joint distribution function (Nelsen, 2006). In reality, copulas are a powerful tool for representing the structure of dependence between random variables and have recently gained attention as an effective method in modeling multivariate phenomena. According to the research of Joe (1997) and Kurowicka and Cooke (2007), multivariate data can be modeled using hierarchical structures based on copulas. For example, suppose a random vector $X = (X_1, ..., X_n)$ exists with a joint density function $f(x_1, ..., x_n)$. This density function can be expressed as follows:

$$f(x_1,...,x_n) = f_n(x_n).f(x_{n-1} \mid x_n).\, f(x_{n-2} \mid x_{n-1},x_n)..f(x_1 \mid x_2,...,x_n) \tag{1}$$

This approach enables the modeling of the most complex dependency relationships and has widespread applications in various fields, including hydrology, finance, and engineering. This decomposition is unique until the reproduction of variables. Each joint distribution function implicitly includes the marginal behavior of the single variables and the structure of their dependence.

A copula provides a way to separate this structure of dependence. A copula is a multivariate distribution C with uniform margins $U(0, 1)$ on [0, 1]. Sklar's theory states that any multivariate distribution F with marginals $F_1(x_1),...,F_2(x_2)$ can be written as follows (Nazeri Tahroudi et al., 2022):

$$f(x_1,...,x_n) = C\{F_1(x_1),...,F_n(x_n)\} \tag{2}$$

For some multi-dimensional (n-dimensional) copulas C, the above relationship is modified as follows:

$$C(u_1,...,u_n) = F\{F_1^{-1}(u_1),...,F_n^{-1}(u_n)\} \tag{3}$$

where $F_n^{-1}(u_n)$ are the inverse of the marginal distribution functions. Given the joint density function f, for continuous, non-decreasing values of F, the marginal densities $F_1, ..., F_2$ are estimated using the Chain law as follows for some n-variate copula densities $c_{1...n}(.)$:

$$f(x_1,...,x_n) = c_{1...n}\{F_1(x_1),...,F_n(x_n)\}.\, f_1(x_1)..f_n(x_n) \tag{4}$$

There are various copulas in the two-variable case. The most important and widely used ones in meteorology, hydrology, and water resources studies are the Clayton, Ali-Mikhail-Haq, Farlie-Gumbel-Morgenstern, Frank, Galambos, Gumbel-Hougaard, and Plackett copulas. The structure of these copulas is as follows (Nelsen, 2006):

Ali-Mikhail-Haq Copula (AMH)

$$C(u,v) = \frac{uv}{1-\theta(1-u)(1-v)} \tag{5}$$

where the range of variation of parameter θ is $-1 \le \theta \le 1$.

Clayton Copula

$$C(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta} \quad (6)$$

where $\theta \geq 0$.

Frank Copula

$$C(u,v) = -\frac{1}{\theta}\ln\left[1 + \frac{\left(e^{-\theta u}-1\right)\left(e^{-\theta v}-1\right)}{e^{-\theta}-1}\right] \quad (7)$$

where $\theta \neq 0$

Galambos Copula

$$C(u,v) = uv\exp\left\{\left[\left(-\ln u\right)^{-\theta} + \left(-\ln v\right)^{-\theta}\right]^{-\frac{1}{\theta}}\right\} \quad (8)$$

where $\theta \geq 0$.

Gumbel-Hougaard Copula (GH)

$$C(u,v) = \exp\left\{-\left[\left(-\ln u\right)^{\theta} + \left(-\ln v\right)^{\theta}\right]^{\frac{1}{\theta}}\right\} \quad (9)$$

where $\theta \geq 0$.

Plackett Copula

$$C(u,v) = \exp\left\{-\left[\left(-\ln u\right)^{\theta} + \left(-\ln v\right)^{\theta}\right]^{\frac{1}{\theta}}\right\} \quad (10)$$

where $\theta \geq 0$.

Farlie-Gumbel-Morgenstern Copula (FGM)

$$C(u,v) = \frac{1}{2}\frac{1}{\theta-1}\left\{\begin{matrix}1+(\theta-1)(u+v)- \\ \left[\left(1+(\theta-1)(u+v)\right)^2 - 4\theta\left(\theta-1\right)uv\right]^{\frac{1}{2}}\end{matrix}\right\} \quad (11)$$

where $-1 \leq \theta \leq 1$.

To find the best fit of the copula distribution, an empirical copula is used. In the two-dimensional case, the empirical copula (u_i, v_i) is as follows:

$$C_e(u_i, v_i) = \frac{1}{n}\sum_{i=1}^{n} I\left(\frac{Q_i}{n+1} \leq u_i, \frac{P_i}{n+1} \leq v_i\right) \quad (12)$$

where C_e is the empirical copula, n is the number of observations, and *I(A)* is the indicator variable of a. If the condition *A* is true, the value is one; otherwise, it is zero. Q_i and P_i is the rank of the observational data *i* related to suspended sediment load and river flow discharge (PronoosSedighi et al., 2022).

2.3. Conditional Density in Two Dimensions

The conditional density function is used to analyze the frequency of suspended sediment load values conditional on the occurrence of river flow discharge. The conditional density in two dimensions is calculated using equation 13 (Khashei et al., 2022):

$$c(u_2 \mid u_1) = \left.\frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2}\right|_{u_1} \quad (13)$$

In this equation, u_1 and u_2 represent the river flow discharge and suspended sediment load values at the station of study, respectively. To obtain a sample u_1 from *d* pairs of variables, the following steps are performed:

$$\begin{aligned} &First: Sample\, w_j \overset{i.i.d.}{\sim} U\left[0;1\right],\ j = 1,\ldots,d \\ &Then: u_1 := w_1 \\ &\quad u_2 := C_{2|1}^{-1}\left(w_2 \mid u_1\right) \\ &\quad \vdots \\ &\quad u_d := C_{d|d-1,\ldots,1}^{-1}\left(w_d \mid u_{d-1},\ldots,u_1\right) \end{aligned} \quad (14)$$

To determine the conditional distribution functions *Cj|j-1,…,1, j = 1…,d,* copula structures are required. This provides a recursive expression using functions *h* for the desired conditional distribution function, which can be easily recursively inverted (Czado, 2019).

2.4. Model evaluation criteria

The Root Mean Square Error (RMSE) and the Nash-Sutcliffe Model Efficiency coefficient (NSE) are the statistics used to select the best copula and also to compare the simulation results (Nash and Sutcliffe, 1970).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(p_i - \overline{p}_i)^2} \quad (15)$$

$$NSE = 1 - \left(\frac{\sum_{i=1}^{N}(p_i - \overline{p}_i)^2}{\sum_{i=1}^{N}(p_i - p_{ave})^2}\right) \quad (16)$$

where $\overline{p}_i$ and p_i are the simulated and observed values, respectively, *N* is the number of data points, and p_{ave} is the mean of the observed values.

3. Results and Discussion

In this study, the aforementioned values were used for the joint frequency analysis of river flow discharge and suspended sediment load at the studied stations (Abajalo and Tapik) in the Nazlochai sub-basin, Lake Urmia, using the available statistical period. The correlation between the studied variables, along with their scatterplot and histograms, are shown in Figure 2 in both the real data and normalized data cases. According to Figure 2, the existence of correlation between the studied variables can be confirmed. The histograms of the studied data also indicate the similarity of the distribution of the studied variables.

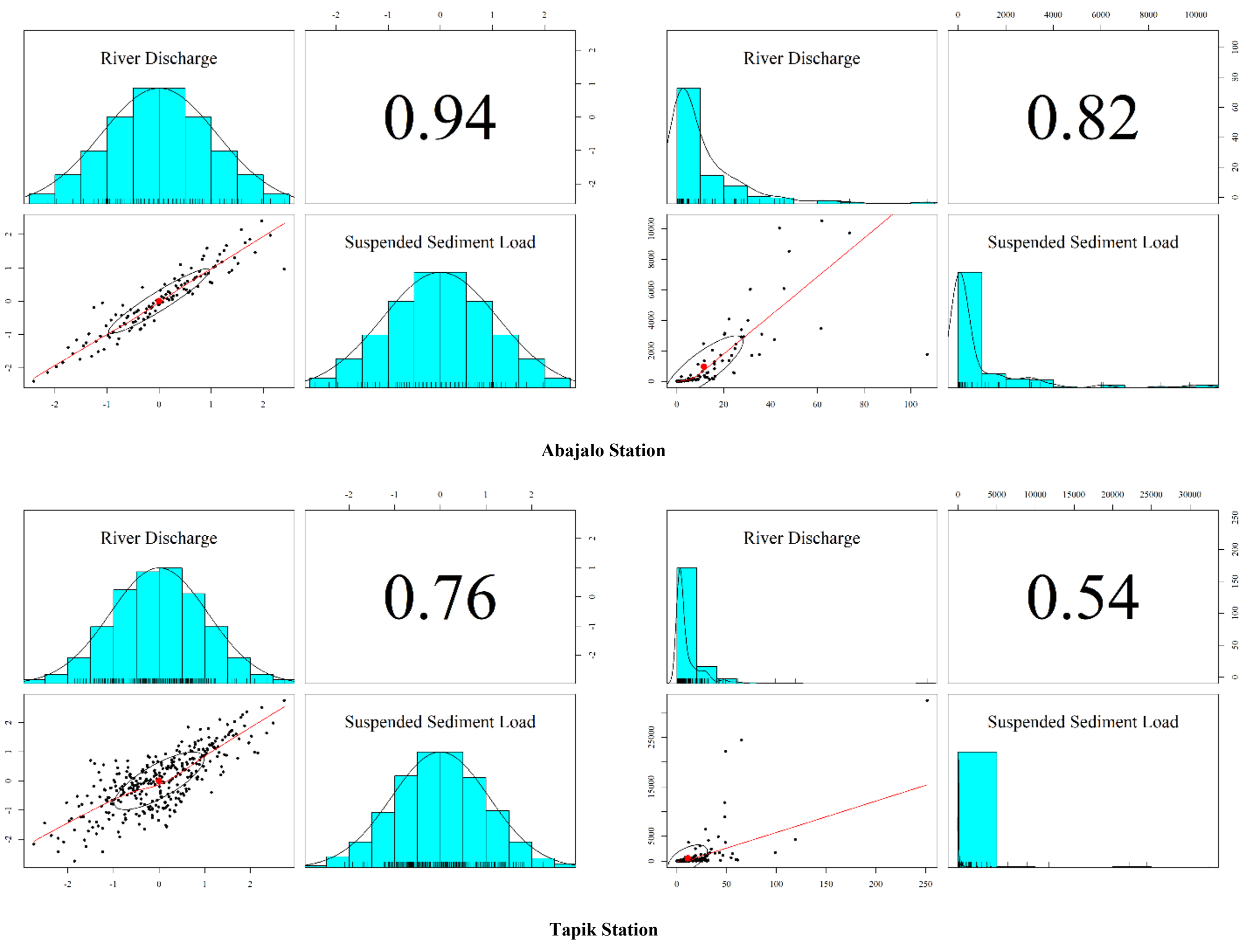


Fig. 2. Correlation, Scatter Plot, and Histograms of initial river flow discharge (m^3/s) and suspended sediment load (ton/day) at studied stations (right: real scale, left: copula scale)

The Kendall's tau correlation coefficient presented in Figure 2, in both studied cases (real and normalized values), showed that the suspended sediment load values have an acceptable and appropriate correlation with the corresponding river flow discharge values at the studied stations. According to Figure 2, it can be concluded that the initial condition for using copula functions in the probability of joint occurrence and two-variable frequency analysis is met.

The Kendall's tau correlation coefficient calculated at both stations indicates a positive and strong correlation between river flow discharge and suspended sediment load. Since Kendall's tau works based on ranking data, it is more suitable for non-normal or outlier-containing data than the Pearson coefficient. A value of 0.82 at the Abajalo station is equivalent to a Spearman or Pearson correlation coefficient of approximately 0.9, indicating the strength of the relationship. The results show that as river flow discharge increases, the amount of suspended sediment also increases significantly. This can be attributed to increased erosion of the riverbed in strong flows.

After confirming the existence of correlation in the studied paired variable at the Abajalo and Tapik stations, the marginal distribution appropriate for the peak river flow discharge and corresponding suspended sediment load data at the aforementioned stations was investigated based on the RMSE and NSE statistics. Common marginal distributions such as Normal, Log-Normal, Exponential, Gamma, Generalized Extreme Value (GEV), Logistic, Log-Logistic, Rayleigh, Nakagami, Generalized Pareto, and Weibull were fitted to the data, and the best ones were selected.

The results of fitting different distributions to the studied paired variable (river flow discharge-suspended sediment load) are presented in Tables 2 and 3. According to Table 2, for river flow discharge values at the Abajalo station, the best distribution is Log-Normal (with NSE=0.99 and RMSE=3.48). Other marginal distributions also showed good performance in fitting the river flow discharge values, for example, Generalized Pareto, Weibull, Log-Logistic, Gamma, and GEV (all NSE > 0.95 and RMSE < 5).

Table 2. Results of fitting marginal distributions to the pair-variable of river flow discharge and suspended sediment load at Abajalo station

Marginal	River flow discharge		Suspended Sediment Load	
	NSE	RMSE (m^3/s)	NSE	RMSE (ton/day)
Normal	0.79	13.14	0.50	20.54
Log-Normal	0.99	**3.48**	0.98	3.63
Exponential	0.81	12.44	-0.14	30.82
Gamma	0.98	4.34	0.91	8.87
GEV	0.98	4.34	**0.99**	**3.36**
Logistic	0.87	10.27	0.65	17.07
Log-Logistic	0.98	3.61	0.98	4.14
Rayleigh	-0.07	29.93	-1.35	44.35
Nakagami	0.95	6.40	0.86	10.98
Generalized Pareto	0.97	4.94	0.98	4.28
Weibull	0.98	3.61	0.96	5.50

Table 3. Results of fitting marginal distributions to the pair-variable of river flow discharge and suspended sediment load at Tapik station

Marginal	River flow discharge		Suspended Sediment Load	
	NSE	RMSE (m^3/s)	NSE	RMSE (ton/day)
Normal	0.69	16.03	0.24	25.14
Log-Normal	0.99	2.91	0.99	2.58
Exponential	0.87	10.50	-0.84	39.19
Gamma	0.94	7.34	0.77	13.83
GEV	0.99	2.23	1.00	1.81
Logistic	0.83	11.74	0.44	21.58
Log-Logistic	0.99	2.83	1.00	1.75
Rayleigh	-0.47	35.08	-2.30	52.45
Nakagami	0.83	11.83	0.63	17.46
Generalized Pareto	0.98	4.03	1.00	1.70
Weibull	0.96	5.81	0.95	6.56

Regarding river flow discharge values, the weakest distribution was Rayleigh (NSE=-0.07 and RMSE=29.93), which performed even worse than the simple average. At the Abajalo station for suspended sediment load, the best distribution is GEV (with NSE=0.99

and RMSE=3.36), which achieved a very good fit with the suspended sediment load data. The performance of other distributions examined in fitting the suspended sediment load data was also relatively good, with Log-Normal (NSE=0.98, RMSE=3.63) and Generalized Pareto (NSE=0.98, RMSE=4.28) being mentioned.

The weakest fit at the Abajalo station for suspended sediment load values is similar to the river flow discharge values and corresponds to the Rayleigh (NSE=-1.35 and RMSE=44.35) and Exponential (NSE=-0.14, RMSE=30.82) distributions, which are completely unsuitable. Overall, the results show that for modeling river flow discharge, Log-Normal or Weibull are very good options. For modeling suspended sediment load, GEV or Log-Normal are considered the best options.

Finally, after selecting the best marginal distributions for both river flow discharge and corresponding suspended sediment load variables at the two stations under investigation, various copulas were evaluated to construct the joint CDF of the studied paired variable. In this study, various copulas including Clayton, Farlie-Gumbel-Morgenstern, Ali-Mikhail-Haq, Galambos, Frank, Plackett, and Gumbel-Hougaard were investigated. The aforementioned copulas were evaluated based on the RMSE and NSE statistics, and the results are presented in Figure 3.

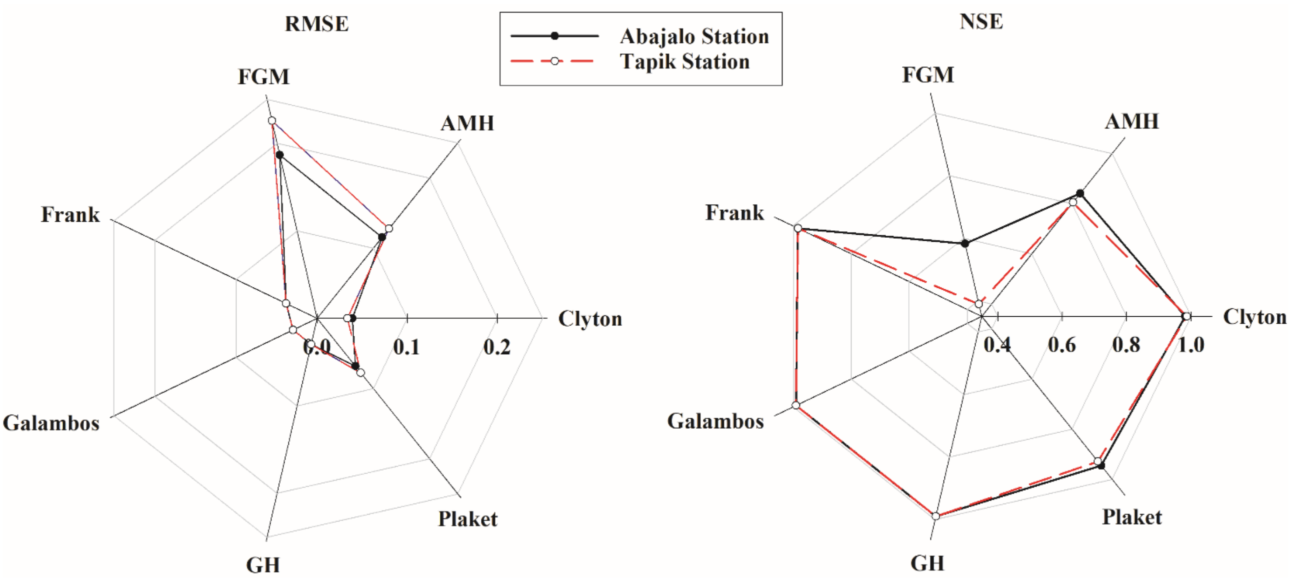


Fig. 3. Results of RMSE and NSE in determining the best copulas in joint frequency analysis of river flow discharge-suspended sediment load at studied stations

According to Figure 3, the best performance at the Abajalo station corresponds to the Galambos and Gumbel-Hougaard copulas with values of NSE≈ 0.989 and RMSE ≈ 0.0296, which have the best fit. The difference between these two is negligible, but the Galambos copula performs better. According to the model evaluation criteria, the Frank and Clayton copulas also perform well (NSE > 0.98, RMSE ≈ 0.038). Among these, the Plackett copula's performance with NSE ≈ 0.945 and RMSE ≈ 0.068 is in the moderate range, and the poorest performance belongs to the AMH and FGM copulas, which are the least accurate.

At the Tapik station as well, consistent with the evaluation statistics in Figure 3, the Galambos and GH copulas still have the best performance (NSE ≈ 0.989, RMSE ≈ 0.0296). The difference between these two functions is negligible (GH is slightly better than Galambos, but the difference is very small). Among the studied copulas, the Clayton copula with NSE ≈ 0.987 and RMSE ≈ 0.033 performs very well and is even better than the Frank copula, and the functions with good performance can be considered suitable alternatives to the selected copulas. In both studied stations, the AMH and FGM copulas still perform poorly and should be eliminated.

According to the results in Figure 3, the Galambos and Gumbel-Hougaard copula functions were selected as the best copula for the joint frequency analysis of the river flow discharge-suspended sediment load paired variable for the Abajalo and Tapik stations, respectively. The copula parameter values for the selected copulas are 15.11 and 98.9, respectively.

3.1. Estimation of the probability of joint occurrence of river flow discharge-suspended sediment load values

After confirming the performance of the best copula at the aforementioned stations and considering the copula coefficient, as well as the marginal distributions appropriate for the river flow discharge and suspended sediment load values at the Abajalo and Tapik stations, the frequency analysis and probability of joint occurrence of these two variables were performed at both stations. The results of the joint frequency analysis of the river flow discharge-suspended sediment load paired variable at the Abajalo and Tapik stations are presented in Figures 4 and 5, respectively.

According to Figure 4, it can be observed that at the Abajalo station, with a probability of more than 80%, conditional on a river flow discharge between 20 and 40 m^3/s, the suspended sediment load at this station will be more than 1000 tons/day. With an increase in probability to more than 90%, conditional on a river flow discharge of more than 40 m^3/s, the suspended sediment load at the Abajalo station will be more than 3000 tons/day.

Considering different probabilities, the corresponding suspended sediment load values for the river flow discharge can be estimated. For a river flow discharge of more than 40 m^3/s and a probability of 90%, the sediment load reaches >3000 tons/day. This sharp increase is consistent with sediment hysteresis models, where sediment particles with higher flow energy are transported exponentially (Williams, 1989). The difference in probabilities (80% and 90%) indicates the model's sensitivity to changes in river flow discharge. For example, in Abajalo, a 10% increase in probability (from 80% to 90%) leads to a threefold increase in sediment load, which can be attributed to the effects of changing flow regime from bedload to suspended load (Knighton, 1999).

At the Tapik station, as shown in Figure 5, if a river flow discharge of 15 to 25 m^3/s occurs, with an 80 percent probability, the suspended sediment load will vary between 200 and 500 tons/day. Considering a probability greater than 90 percent, conditional on a river flow discharge exceeding 25 m^3/s, the suspended sediment load will be greater than 500 tons/day. A sediment load of 500-200 tons/day has been predicted with an 80% probability for river flow discharge values of 25–15 m^3/s. This lower range compared to Abajalo may be due to differences in watershed characteristics (such as slope, vegetation cover, or bed material) (Turowski et al., 2010). For river flow discharge values greater than 25 m^3/s, a sediment load of more than 500 tons/day occurs with a 90% probability. These results are consistent with the studies of Van Rijn (1984), which indicate that after a river flow discharge exceeds a threshold (e.g., 25 m^3/s), the sediment transport capacity increases sharply due to increased bed shear stress.

Numerous studies have shown that there is a direct and non-linear relationship between river flow discharge and suspended sediment load. Particularly at high river flow discharge values, an increase in discharge leads to an exponential increase in sediment load (Walling and Webb, 1996; Syvitski et al., 2000). The use of probabilistic approaches (such as Copula or Bayesian models) is recommended for predicting sediment load, especially in watersheds with incomplete data (Zhang and Singh, 2012). On the other hand, the results indicate that at high river flow discharge values (40 m^3/s in Abajalo), the risk of sedimentation in reservoirs or the blockage of waterways increases.

This finding is consistent with the studies of Kondolf et al. (2014) regarding the impact of sediment on the lifespan of dams. The results show that river flow discharge is a key factor in determining sediment load, but its relationship is non-linear and dependent on watershed characteristics. The use of probabilistic models (such as Copula) can increase the accuracy of predictions. For sustainable management, the development of sediment reduction programs tailored to each station is essential.

3.2. Conditional probability estimation

In the next step, to estimate the conditional probability of occurrence of the paired variables of interest at the Abajalo and Tapik stations, the conditional density of the selected copula functions was used. The results of the conditional probability estimation are presented in Figure 6. In this figure, probabilities greater than 80 percent are considered. According to Figure 6, different suspended sediment load values can be estimated with respect to different river flow discharge values with probabilities of 80-90 percent, 90-95 percent, and 95-99 percent. For example, at the Abajalo station, if a river flow discharge of approximately 30 m^3/s occurs in the study area, the suspended sediment load will be approximately 3500, 6000, and 7000 tons/day with probabilities of 80-90 percent, 90-95 percent, and 95-99 percent, respectively. With the river flow discharge values in the study area with different probabilities, the corresponding suspended sediment load values can be estimated. This curve can be used as a typical curve at the studied stations for bivariate and conditional estimation of suspended sediment load values, conditional on the occurrence of river flow discharge.

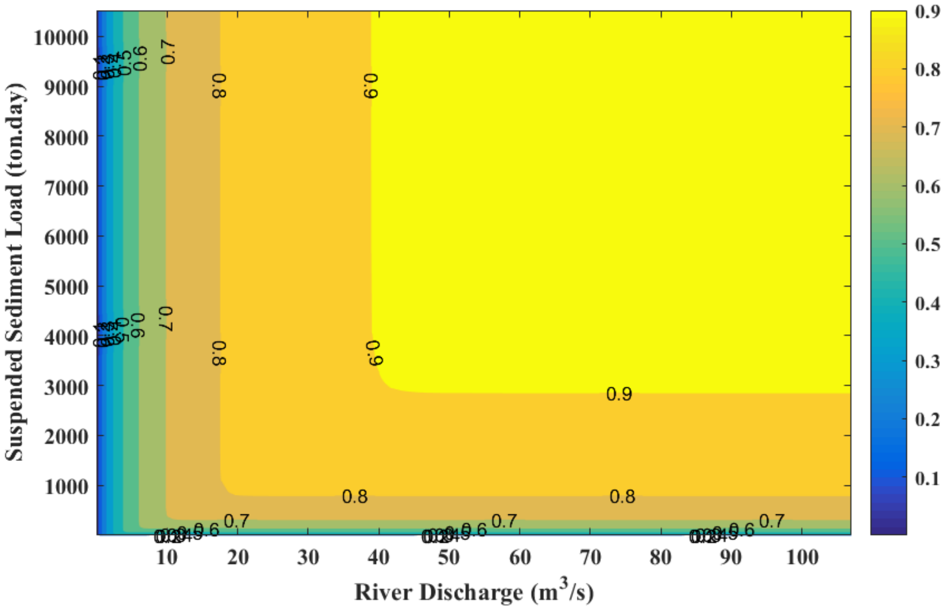


Fig. 4. Results of estimating the joint probability of river flow discharge-suspended sediment load values at the Abajalo station.

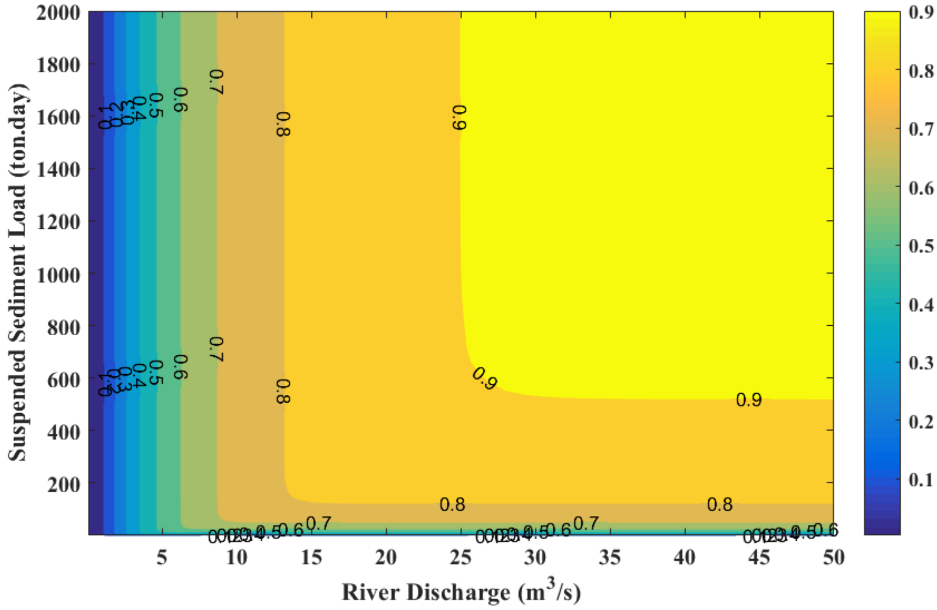
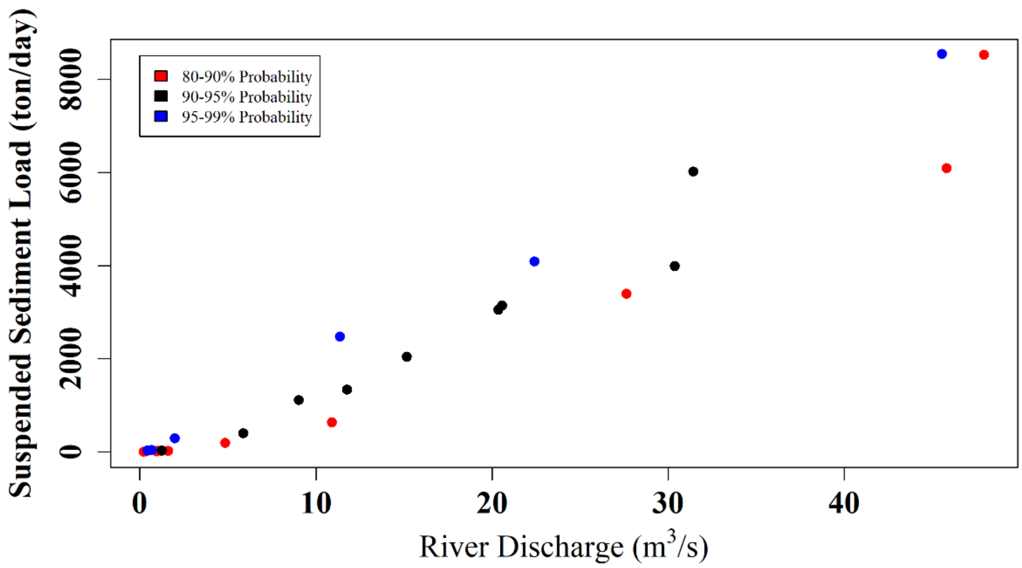
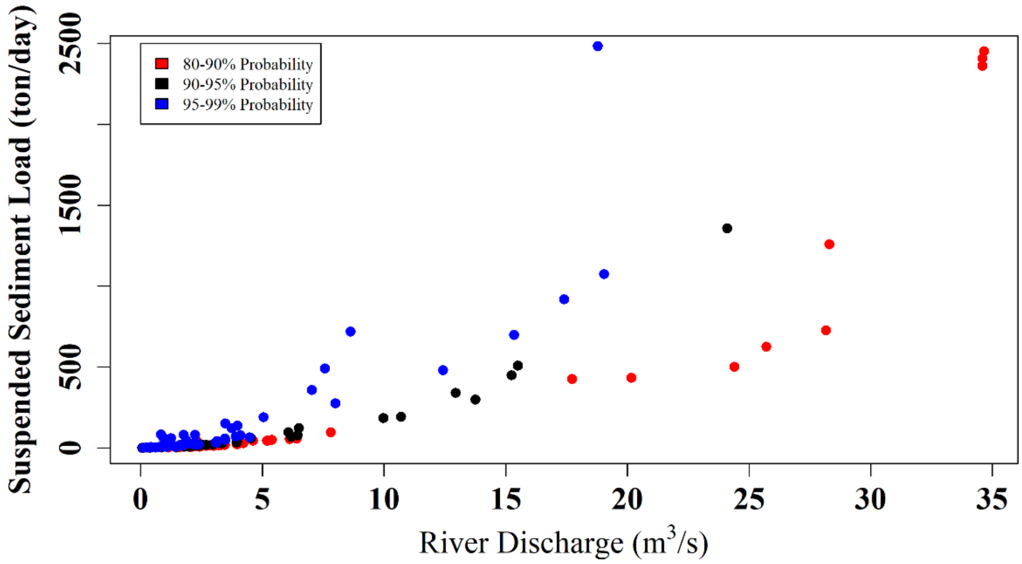


Fig. 5. Results of estimating the joint probability of river flow discharge-suspended sediment load values at the Tapik station.



a



b

Fig. 6. Probability of occurrence of various suspended sediment load values conditional on the occurrence of corresponding river flow discharge at a: Abajalo station and b: Tapik station with probabilities greater than 80 percent

According to Figure 6 and with the availability of river flow discharge values at the studied stations, the corresponding suspended sediment load values can be estimated with high accuracy and probabilities greater than 80 percent. Furthermore, as observed in Figure 6, with an increase in the probability level, the estimated suspended sediment load values at a specific river flow discharge increase. Considering existing studies on climate change and rainfall distribution (Khalili et al., 2016), it can be observed that the persistence of extreme rainfall events has increased, which disrupts the soil structure and consequently results in an increase in river sediment load. An increase in river sediment load reduces the lifespan of water storage structures such as dam reservoirs downstream. Given the current conditions, the need for surface water resource management is felt more than ever. Figure 6 shows each probability of occurrence, representing a range of changes in suspended sediment load with different probabilities, which is only possible under joint and multi-variable conditions. This advantage is also mentioned in the research of PronoosSedighi et al. (2022).

Considering the probabilities presented at different levels, a proposed relationship for estimating suspended sediment load (tons/day) at the studied station locations conditional on river flow discharge (m^3/s) can be provided. The proposed relationships with different probabilities are as follows:

Proposed equation for estimating suspended sediment load conditional on river flow discharge with different probabilities at the Abajalo station:

80% to 90% probability $SSL = 2.6401Q^{1.6644}$

90% to 95% probability $SSL = 3.8166Q^{1.6121}$

95% to 99% probability $SSL = 11.492Q^{1.6234}$

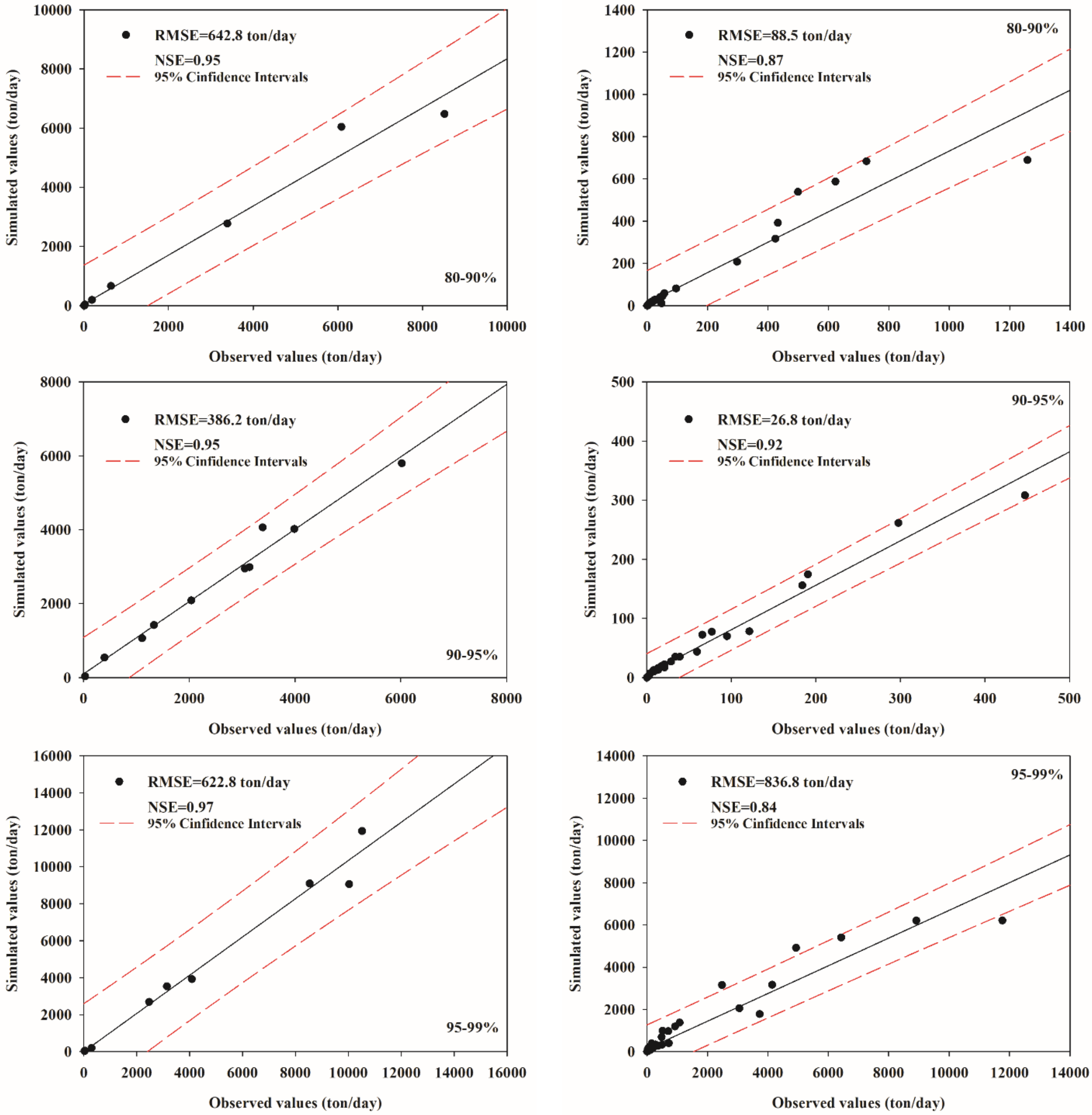
Proposed equation for estimating suspended sediment load conditional on river flow discharge with different probabilities at the Tapik station:

80% to 90% probability $SSL = 16.724Q^{1.5399}$

90% to 95% probability $SSL = 26.143Q^{1.5552}$

95% to 99% probability $SSL = 81.585Q^{1.2457}$

The accuracy and performance of the proposed relationships were evaluated using available data. The results of simulating suspended sediment load values using the proposed relationships are presented in Figure 7.



a *b*

Fig. 7. Results of simulating suspended sediment load values using the proposed relationships (a: Abajalo station and b: Tapik station)

According to Figure 7-a, it can be observed that the performance of the proposed relationships in simulating suspended sediment load values at the Abajalo station at probabilities of 80-90, 90-95, and 95-99 percent shows performance of 95, 95, and 97 percent, respectively, based on the NSE index. These results indicate that the proposed relationships have a high ability to predict suspended sediment load at all probability levels. The proposed relationship with the mentioned probabilities has errors of 8.642, 3.86, and 8.622 tons/day, respectively. The error of the proposed relationship at the 90-95 percent probability is lower than other probabilities under investigation. This indicates that the model performs optimally within this probability range.

Furthermore, according to the 95 percent confidence interval simulation, all simulated values are located within the desired range, and neither underestimation nor overestimation is observed in any of the investigated probabilities (80-90, 90-95, and 95-99) at the Abajalo station. The absence of underestimation or overestimation within the 95% confidence interval indicates model stability (Nazeri Tahroudi et al., 2025).

According to Figure 7-b, it can also be observed that the performance of the proposed relationships in simulating suspended sediment load values at the Tapik station at probabilities of 80-90, 90-95, and 95-99 percent shows performance of 87, 92, and 84 percent, respectively. The proposed relationship estimated errors of 8.88, 2.26, and 8.836 tons/day, respectively, with the mentioned probabilities. NSE values at the Tapik station are estimated to be lower than at the Abajalo station.

The 1-1 lines at the Abajalo station are closer to the simulated values compared to the Tapik station. At the Tapik station, at the 90-95 percent probability, the simulated values are within the 95 percent confidence interval, but the relationships presented in the 80-90 and 99-95 percent confidence intervals have, in some cases, simulated values outside the confidence interval. This was not observed at the Abajalo station.

The proposed relationships, due to being estimated within a specific confidence interval based on copulas functions, have higher accuracy compared to the usual case. Frequency analysis of two variables, river flow discharge and suspended sediment load, in each region can be a suitable tool for understanding the joint occurrence of these two phenomena and can provide better results than single-variable models. Watershed management and planning require the application of these methods in practical studies.

4. Conclusion

This study provided a detailed analysis of the joint relationship between river flow discharge and suspended sediment load in the Nazlochai basin using copula functions and marginal distributions. Key findings showed a strong correlation between the input variables (river flow discharge and corresponding suspended sediment load). Kendall's tau coefficient (0.82 at the Abajalo station) confirmed a strong non-linear relationship between river flow discharge and suspended sediment load, particularly at high river flow discharge values where an exponential increase in sediment load was observed. Similar results were obtained for the Tapik station.

Regarding marginal distributions, the best distributions based on NSE and RMSE criteria were selected for the paired variable of interest (river flow discharge-suspended sediment load) at both stations. Examining the copula functions in the joint frequency analysis of the river flow discharge-suspended sediment load variable pair showed that the Galambos and Gumbel-Hougaard copula functions were suitable for modeling the joint variables with the least error (RMSE≈0.0296) and highest accuracy (NSE≈0.989). Examining the joint probabilities of river flow discharge-suspended sediment load at the studied stations showed that at the Abajalo station, a river flow discharge of 40 m^3/s with a 90% probability leads to a suspended sediment load of more than 3000 tons/day, while at the Tapik station, a river flow discharge of 25 m^3/s with the same probability creates a suspended sediment load of more than 500 tons/day.

Finally, considering the copula functions, marginal distributions, and the conditional density of the copula functions, conditional simulation relationships for suspended sediment load conditional on river flow discharge at different probability levels were presented. Based on the estimated probabilities, a relationship for estimating conditional suspended sediment load based on river flow discharge was developed. The NSE efficiency coefficient confirmed the effectiveness of the proposed relationships in simulating suspended sediment load at the studied stations.

In this study, an approach is proposed that can estimate the amount of suspended sediment load at the Abajalo and Tapik stations with probabilities greater than 80 percent, by providing probabilistic curves and simulation relationships with different probabilities. This method is localized because it uses statistical distributions of data and the conditional density of copula functions appropriate to the statistical distribution, and therefore the proposed method has no geographical or climatic limitations. This approach can be used and beneficial in managing suspended sediment load, considering the increase in extreme rainfall and recent floods, by providing specific charts for each region.

5. Conflict of interest

No potential conflict of interest was reported by the authors.

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